An exactly solvable dissipative transport model

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Abstract
We introduce a class of one-dimensional lattice models in which a quantity, which may be thought of as an energy, is either transported from one site to a neighbouring one, or locally dissipated. Transport is controlled by a continuous bias parameter, q, which allows us to study symmetric as well as asymmetric cases. We derive sufficient conditions for the factorization of the N-body stationary distribution and give an explicit solution for the latter, before briefly discussing physically relevant situations.

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Introduction

Systems driven in a non-equilibrium steady state by an external force that generates an internal flux are very frequent in nature. Among these systems, two broad classes may be distinguished according to their conservation properties. The first class corresponds to systems in which the flux of a conserved quantity (e.g., particles or mass) takes place. Such systems have raised considerable interest recently, and different paradigmatic models for which the full (N-body) stationary probability distribution can be worked out exactly have emerged from these studies: the asymmetric simple exclusion process (ASEP) [1–5], the zero range process (ZRP) [1, 6, 7], the asymmetric random average process (ARAP) [8–11], as well as more general mass transport models [12, 13].

On the other side, a second class corresponds to situations where the quantity moving through the system is locally non-conserved in the bulk. This may happen, for instance, when the system can exchange particles with a reservoir, or when one considers an energy flux in a system where the 'microscopic' dynamics—at the chosen level of description—is already dissipative, as in turbulent flows or in shaken granular materials, for instance. Within this class of systems, attention has been mainly devoted to ASEP or ZRP models where particles can be added or removed within the bulk, at rates that may differ from the boundary ones [7, 14–18]. Yet there seems to be very few known solvable models, for which the probability of any microscopic configuration can be computed exactly, where a continuous quantity is
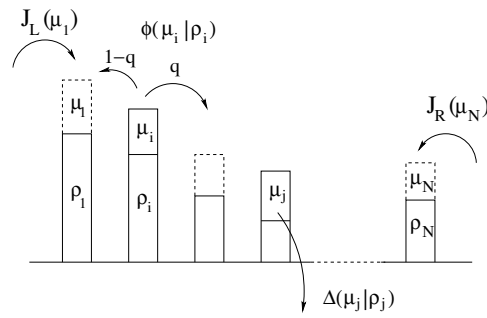
Figure 1. Schematic view of the model, with the three different mechanisms: energy injection at boundary sites with rates $J_L(\mu_1)$ and $J_R(\mu_N)$, transport from site i to site $i + 1$ and $i - 1$ with rates $q\phi_i(\mu_i|\rho_i)$ and $(1 - q)\phi_i(\mu_i|\rho_i)$ respectively, and dissipation at site j with rate $\Delta_j(\mu_j|\rho_j)$. On sites $i = 1$ and $i = N$, energy may also be transferred to the reservoirs; see text.

injected at the boundaries and (partially) dissipated in the bulk, a situation of broad physical interest [19][1].

In this paper, we consider a generalization of the one-dimensional cascade model introduced in [20], where a local quantity, injected at the boundaries, is either transported or dissipated in the bulk. A bias parameter q allows us to consider partially asymmetric transport. We give an explicit solution for the full N-body stationary distribution in cases where it factorizes, and briefly discuss a physical application to a schematic turbulence modelling.

Definition of the model

We consider a one-dimensional lattice with sites labelled from $i = 1$ to N, and introduce a local quantity ρ_i which may be thought of as an energy. The dynamics is defined through the following local stochastic rules. An amount of energy between μ and $\mu + \mathrm{d}\mu$ can be injected from a reservoir on the left boundary $i = 1$ with a rate (probability per unit time) $J_L(\mu)\,\mathrm{d}\mu$, or injected from another reservoir on the right boundary $i = N$ with a rate $J_R(\mu)\,\mathrm{d}\mu$. Transport proceeds by removing an amount of energy μ on site i with a rate $\phi_i(\mu|\rho_i)\,\mathrm{d}\mu$ depending on the energy ρ_i present on site i. The energy μ is then added either to site $i + 1$ (to the right reservoir if $i = N$) with probability q, or to site $i - 1$ (to the left reservoir if $i = 1$) with the complementary probability $p = 1 - q$. Finally, dissipation is implemented by withdrawing an energy μ from site i with a rate $\Delta_i(\mu|\rho_i)\,\mathrm{d}\mu$, without adding it to another site. A simple sketch of the model is shown in figure 1.

The statistical state of the model is described by the N-body probability distribution $P(\rho_1, \ldots, \rho_N, t)$. Given the above stochastic rules, the evolution of this distribution is governed by the following master equation:

$$\frac{\partial P}{\partial t} = q \sum_{j=1}^{N-1} \int_0^{\rho_{j+1}} \mathrm{d}\mu \, \phi_j(\mu|\rho_j + \mu) P(\ldots, \rho_j + \mu, \rho_{j+1} - \mu, \ldots, t)$$

$$+ p \sum_{j=2}^{N} \int_0^{\rho_{j-1}} \mathrm{d}\mu \, \phi_j(\mu|\rho_j + \mu) P(\ldots, \rho_{j-1} - \mu, \rho_j + \mu, \ldots, t)$$

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[1] Note that a 'reverse' situation, where a quantity is constantly added in the bulk, has been studied in the context of force fluctuations in beads packs, the vertical axis playing the role of time [10].

$$+ \sum_{j=1}^{N} \int_0^{\infty} d\mu \, \Delta_j(\mu|\rho_j + \mu) P(\ldots, \rho_j + \mu, \ldots, t)$$

$$+ p \int_0^{\infty} d\mu \, \phi_1(\mu|\rho_1 + \mu) P(\rho_1 + \mu, \ldots, t)$$

$$+ q \int_0^{\infty} d\mu \, \phi_N(\mu|\rho_N + \mu) P(\ldots, \rho_N + \mu, t)$$

$$+ \int_0^{\rho_1} d\mu \, J_L(\mu) P(\rho_1 - \mu, \ldots, t) + \int_0^{\rho_N} d\mu \, J_R(\mu) P(\ldots, \rho_N - \mu, t)$$

$$- \sum_{j=1}^{N} \int_0^{\rho_j} d\mu [\phi_j(\mu|\rho_j) + \Delta_j(\mu|\rho_j)] P(\{\rho_i\}, t)$$

$$- \int_0^{\infty} d\mu [J_L(\mu) + J_R(\mu)] P(\{\rho_i\}, t),$$

$$(1)$$

where the dots stand for all the variables ρ_i which are not modified by μ.

Factorized steady state

We wish to study the stationary distribution $P_{\mathrm{st}}(\rho_1, \ldots, \rho_N)$ in cases where it is factorized, and thus make the following ansatz for $P_{\mathrm{st}}(\rho_1, \ldots, \rho_N)$:

$$P_{\mathrm{st}}(\rho_1, \ldots, \rho_N) = \frac{1}{Z} \prod_{i=1}^{N} f_i(\rho_i), \tag{2}$$

where Z is a normalization factor. Note that due to the presence of dissipation in the bulk, one expects that the one-site distribution $f_i(\rho_i)$ is different from one site to the other, even when the stochastic rules are site-independent, i.e., $\phi_i(\mu|\rho_i) = \phi(\mu|\rho_i)$ and $\Delta_i(\mu|\rho_i) = \Delta(\mu|\rho_i)$. Introducing the above ansatz for $P_{\mathrm{st}}(\rho_1, \ldots, \rho_N)$ in the stationary master equation, it is easy to check that a specific form for the transport and dissipation functions $\phi_i(\mu|\rho_i)$ and $\Delta_i(\mu|\rho_i)$ allows for a decoupling of the different degrees of freedom, in the sense that the stationary master equation can then be written as a sum of terms involving at most one variable ρ_i. This specific form reads

$$\phi_i(\mu|\rho_i) = \tilde{\phi}_i(\mu) \frac{f_i(\rho_i - \mu)}{f_i(\rho_i)}, \qquad \Delta_i(\mu|\rho_i) = \tilde{\Delta}_i(\mu) \frac{f_i(\rho_i - \mu)}{f_i(\rho_i)}. \tag{3}$$

Under these assumptions, the stationary master equation can be rewritten as

$$\sum_{j=1}^{N} \int_0^{\rho_j} d\mu [\tilde{\phi}_j(\mu) + \tilde{\Delta}_j(\mu)] \frac{f_j(\rho_j - \mu)}{f_j(\rho_j)} + \int_0^{\infty} d\mu [J_L(\mu) + J_R(\mu)]$$

$$= q \sum_{j=2}^{N} \int_0^{\rho_j} d\mu \, \tilde{\phi}_{j-1}(\mu) \frac{f_j(\rho_j - \mu)}{f_j(\rho_j)} + p \sum_{j=1}^{N-1} \int_0^{\rho_j} d\mu \, \tilde{\phi}_{j+1}(\mu) \frac{f_j(\rho_j - \mu)}{f_j(\rho_j)}$$

$$+ \int_0^{\infty} d\mu \left(q\tilde{\phi}_N(\mu) + p\tilde{\phi}_1(\mu) + \sum_{j=1}^{N} \tilde{\Delta}_j(\mu) \right)$$

$$+ \int_0^{\rho_1} d\mu \, J_L(\mu) \frac{f_1(\rho_1 - \mu)}{f_1(\rho_1)} + \int_0^{\rho_N} d\mu \, J_R(\mu) \frac{f_N(\rho_N - \mu)}{f_N(\rho_N)} \tag{4}$$

and the decoupling property mentioned above appears explicitly. As this equation is a sum of functions of independent variables, each of these functions—i.e., the sum of terms depending on a given ρ_i—has to be equal to a constant. To determine the value of these constants, one can first send all the $\rho_i's$ to zero, in which case all the integrals over the interval $[0, \rho_i]$ are expected to vanish[2]. As a result, the terms that do not depend on ρ_i sum up to zero:

$$\int_0^\infty d\mu \left(q\tilde{\phi}_N(\mu) + p\tilde{\phi}_1(\mu) + \sum_{j=1}^N \tilde{\Delta}_j(\mu) - J_L(\mu) - J_R(\mu) \right) = 0. \tag{5}$$

In a similar way, sending to zero all the ρ_i but one leads to the conclusion that all terms depending on a given ρ_j also sum up to zero. For $2 \leqslant j \leqslant N - 1$, one has

$$\int_0^{\rho_j} d\mu [q\tilde{\phi}_{j-1}(\mu) + p\tilde{\phi}_{j+1}(\mu) - \tilde{\phi}_j(\mu) - \tilde{\Delta}_j(\mu)] \frac{f_j(\rho_j - \mu)}{f_j(\rho_j)} = 0. \tag{6}$$

At the boundary sites $j = 1$ and $j = N$, one finds

$$\int_0^{\rho_1} d\mu [J_L(\mu) + p\tilde{\phi}_2(\mu) - \tilde{\phi}_1(\mu) - \tilde{\Delta}_1(\mu)] \frac{f_1(\rho_1 - \mu)}{f_1(\rho_1)} = 0 \tag{7}$$

$$\int_0^{\rho_1} d\mu [J_R(\mu) + q\tilde{\phi}_{N-1}(\mu) - \tilde{\phi}_N(\mu) - \tilde{\Delta}_N(\mu)] \frac{f_N(\rho_N - \mu)}{f_N(\rho_N)} = 0. \tag{8}$$

A sufficient condition to solve this set of equations is to assume that each of the integrands is equal to zero, leading to

$$q\tilde{\phi}_N(\mu) + p\tilde{\phi}_1(\mu) + \sum_{j=1}^N \tilde{\Delta}_j(\mu) - J_L(\mu) - J_R(\mu) = 0 \tag{9}$$

$$q\tilde{\phi}_{j-1}(\mu) + p\tilde{\phi}_{j+1}(\mu) - \tilde{\phi}_j(\mu) - \tilde{\Delta}_j(\mu) = 0 \qquad (2 \leqslant j \leqslant N - 1) \tag{10}$$

$$J_L(\mu) + p\tilde{\phi}_2(\mu) - \tilde{\phi}_1(\mu) - \tilde{\Delta}_1(\mu) = 0 \tag{11}$$

$$J_R(\mu) + q\tilde{\phi}_{N-1}(\mu) - \tilde{\phi}_N(\mu) - \tilde{\Delta}_N(\mu) = 0. \tag{12}$$

Thus, as a consequence of the specific form (3) chosen for $\phi_j(\mu|\rho)$ and $\Delta_j(\mu|\rho)$, it turns out that the function $f_j(\rho)$ has disappeared from the equations. Although this result may be surprising at first sight, it should be noted that the mass transport model introduced in [12] has the same property. As a result, any set of functions $f_j(\rho)$ may be a solution of the model, provided that the dynamical rules are suitably chosen. Equations (9)–(12) should thus be considered as compatibility conditions allowing for the existence of a factorized distribution. Let us also note that these four equations are not independent: it can be checked easily that, for instance, equation (9) can be obtained from the sum of equations (10), summed over j, (11) and (12). To solve these equations, we first define some extra functions $\tilde{\phi}_0(\mu)$ and $\tilde{\phi}_{N+1}(\mu)$ on the boundaries as, assuming $p, q \neq 0$,

$$J_L(\mu) \equiv q\tilde{\phi}_0(\mu), \qquad J_R(\mu) \equiv p\tilde{\phi}_{N+1}(\mu), \tag{13}$$

which allows one to rewrite equations (11) and (12) in the same form as equation (10) with $j = 1$ and $j = N$, respectively. Introducing an auxiliary function $\chi_j(\mu)$ through

$$\chi_j(\mu) \equiv \tilde{\phi}_{j+1}(\mu) - \tilde{\phi}_j(\mu), \qquad 0 \leqslant j \leqslant N \tag{14}$$

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[2] Considering an integral of the form $I(\rho) = \int_0^\rho d\mu \, \psi(\mu) f(\rho - \mu)/f(\rho)$ with $\psi(\mu) \sim \mu^{\alpha-1}$ and $f(\mu) \sim \mu^{\beta-1}$ for $\mu \to 0$ ($\alpha, \beta > 0$ to ensure the convergence of the integral), it can be shown easily that $I(\rho) \sim \rho^\alpha \to 0$ when $\rho \to 0$.

equations (10), (11) and (12) can be rewritten in a concise form as

$$p\chi_j(\mu) - q\chi_{j-1}(\mu) = \tilde{\Delta}_j(\mu), \qquad 1 \leqslant j \leqslant N. \tag{15}$$

Setting $r \equiv p/q$, this last equation is easily integrated out as

$$\chi_j(\mu) = \tilde{K}(\mu)r^{N-j} - \frac{1}{q}\sum_{i=j}^{N-1} r^{i-j}\tilde{\Delta}_{i+1}, \qquad 0 \leqslant j \leqslant N, \tag{16}$$

where $\tilde{K}(\mu)$ is (up to now) an arbitrary function of μ, and with the convention that the sum is zero if $j = N$. Using equation (14) and assuming $q \neq 1/2$, one can now solve for $\tilde{\phi}_j(\mu)$, yielding for $0 \leqslant j \leqslant N + 1$ (the sum is zero if $j \geqslant N$)

$$\tilde{\phi}_j(\mu) = K(\mu)r^{N-j} + \frac{1}{2q-1}\sum_{i=j+1}^{N}(1 - r^{i-j})\tilde{\Delta}_i + C(\mu) \tag{17}$$

with a new (arbitrary) function $C(\mu)$, and where $K(\mu)$ is related to $\tilde{K}(\mu)$ through $K(\mu) = p\tilde{K}(\mu)/(2q - 1)$. Using the boundary conditions (13), one can determine the unknown functions $K(\mu)$ and $C(\mu)$:

$$K(\mu) = Br\left(\frac{J_R(\mu)}{1-q} - \frac{J_L(\mu)}{q} + \frac{1}{2q-1}\sum_{i=1}^{N}(1 - r^i)\tilde{\Delta}_i(\mu)\right) \tag{18}$$

$$C(\mu) = \frac{B}{q}\left(J_L(\mu) - J_R(\mu)r^N - \frac{q}{2q-1}\sum_{i=1}^{N}(1 - r^i)\tilde{\Delta}_i(\mu)\right) \tag{19}$$

with $B \equiv (1 - r^{N+1})^{-1}$. Note that the case $q = 1$, which was temporarily excluded from the calculation, is recovered by taking the limit $q \rightarrow 1$. A similar calculation can be performed in the case $q = 1/2$ to give

$$\tilde{\phi}_j(\mu) = 2\sum_{i=1}^{j}(j - i)\tilde{\Delta}_i(\mu) + jA(\mu) + 2J_L(\mu) \qquad 1 \leqslant j \leqslant N \tag{20}$$

with

$$A(\mu) = \frac{2}{N+1}\left[J_R(\mu) - J_L(\mu) - \sum_{i=1}^{N}(N + 1 - i)\tilde{\Delta}_i(\mu)\right]. \tag{21}$$

As mentioned in the above derivation, the form (3) of the rates $\phi_i(\mu|\rho_i)$ and $\Delta_i(\mu|\rho_i)$, as well as the conditions given in equations (17) and (20) are *a priori* only sufficient conditions for the factorization of the distribution $P_{st}(\rho_1, \ldots, \rho_N)$. Yet, let us recall that for the general class of mass transport models on a ring geometry studied in [12, 13], where dissipation is absent, the form (3) of the transport rate has been shown to be a necessary and sufficient condition for factorization, in the case of continuous time dynamics. So it may be plausible that this form is also necessary in the present model. Let us also note that an explicit test of the form (3) of the rate functions has been proposed in [13]. This test should also apply to the present situation so that, given two functions $\phi_i(\mu|\rho_i)$ and $\Delta_i(\mu|\rho_i)$, one should be able to check whether they are of the form (3), and to determine the corresponding functions $\tilde{\phi}_i(\mu)$, $\tilde{\Delta}_i(\mu)$ and $f_i(\rho)$.

Site-independent rates

A situation of physical interest is when the dynamical rules are site-independent whereas, due to the injection and dissipation mechanisms, the one-site steady-state distribution may depend on the site considered. An example of such a situation has been given in [20], in the fully biased case $q = 1$. In what follows, we wish to study the possibility to find a factorized steady-state distribution with symmetric transport, that is for $q = 1/2$, and with site-independent rates. The transition rates given in equation (3) are independent of the site if the functions $\tilde{\phi}_j(\mu)$, $\tilde{\Delta}_j(\mu)$ and $f_j(\rho)$ satisfy

$$\tilde{\phi}_j(\mu) = h_1(\mu)\, e^{-\lambda_j \mu}, \qquad \tilde{\Delta}_j(\mu) = h_2(\mu)\, e^{-\lambda_j \mu}, \qquad f_j(\rho) = g(\rho)\, e^{-\lambda_j \rho}. \tag{22}$$

Besides, from equation (15), the rates have to satisfy

$$\tfrac{1}{2}\tilde{\phi}_{j-1}(\mu) + \tfrac{1}{2}\tilde{\phi}_{j+1}(\mu) - \tilde{\phi}_j(\mu) = \tilde{\Delta}_j(\mu), \qquad 1 \leqslant j \leqslant N. \tag{23}$$

Let us introduce a coordinate $x = j/L$, so that $0 \leqslant x \leqslant 1$. For the sake of simplicity, we shall consider the limit of large system size, and assume that the transition rates vary slowly as a function of the site index, in the sense that there exist continuous functions $\hat{\phi}(\mu, x)$ and $\hat{\Delta}(\mu, x)$ of the variable x, such that

$$\tilde{\phi}_j(\mu) = \hat{\phi}(\mu, jL), \qquad \tilde{\Delta}_j(\mu) = \frac{1}{L^2}\hat{\Delta}(\mu, jL). \tag{24}$$

The $1/L^2$ factor has been included so that equation (23) admits a consistent continuous limit. From equation (22), this means that λ_j must be replaced by a continuous function $\lambda(x)$. Accordingly, $\hat{\phi}(\mu, x)$ and $\hat{\Delta}(\mu, x)$ satisfy the following equation

$$\frac{\partial^2 \hat{\phi}}{\partial x^2}(\mu, x) = 2\hat{\Delta}(\mu, x) \tag{25}$$

which may be rewritten as

$$\mu \lambda'(x)^2 - \lambda''(x) = \frac{2 h_2(\mu)}{\mu h_1(\mu)}. \tag{26}$$

As the rhs of this last equation does not depend on x, the only possibility is that $\lambda(x)$ is linear in x, so that the lhs is also a constant. As a result, it turns out that there is no factorized solution that would be symmetric in x with respect to $x = 1/2$. This suggests that the present model, with $q = 1/2$ and symmetric injection rates $J_L(\mu) = J_R(\mu)$, does not admit a factorized steady state, at least in the continuous limit considered here. The above calculation, with $\lambda(x)$ linear in x, rather corresponds to a cascade process, where energy is injected on one boundary only, in a way essentially similar to the fully biased case [20].

A simple physical application

Coming back to the value $q = 1$ (fully biased model), one can try to use an inhomogeneous version of the model (i.e., with site-dependent rates) in order to describe schematically a turbulent cascade—note that only a homogeneous case was studied in [20]. To this aim, we assume that the lattice sites n may be thought of as successive wavenumbers $k_n = n k_1$, and that energy is injected on site $n = 1$ (corresponding to the largest wavelength) with a rate $J_L(\mu) = J_0\, e^{-\beta\mu}$, whereas there is no energy transfer starting from site $n = N$, that is $J_R(\mu) = 0$ and $\phi_N(\mu|\rho) = 0$. Note that the energy flux entering the system, which in steady state equals the total rate of energy dissipation, is given by $\varepsilon = J_0/\beta^2$. To proceed further, the properties of the transition rates have to be specified. For simplicity, we assume $f_n(\rho)$ to take an exponential form, namely $f_n(\rho) = e^{-\alpha_n \rho}$. A characteristic feature of the turbulence

phenomena is that the energy dissipation rate Ω_n on site n is of the form $\Omega_n = \nu k_n^2 \langle \rho_n \rangle$, where ν is a viscosity. We thus compute Ω_n in the present model, yielding

$$\Omega_n = \frac{1}{Z_n} \int_0^\infty \mathrm{d}\rho \, f_n(\rho) \int_0^\rho \mathrm{d}\mu \, \mu \Delta_n(\mu|\rho) = \int_0^\infty \mathrm{d}\mu \, \mu \tilde{\Delta}_n(\mu) \tag{27}$$

with $Z_n \equiv \int_0^\infty \mathrm{d}\rho \, f_n(\rho)$, and where the last equality is obtained by permuting the integrals and using the form (3). A second constraint arises from equations (17) to (19) encoding the factorization condition for the probability distribution, which reads

$$J_L(\mu) = \sum_{n=1}^N \tilde{\Delta}_n(\mu). \tag{28}$$

A simple form for $\tilde{\Delta}_n(\mu|\rho)$ that matches the above constraints is

$$\tilde{\Delta}_n(\mu) = \frac{\nu k_1^2 n^2}{\alpha_n} \beta^2 \, e^{-\beta\mu} \tag{29}$$

on condition that the following self-consistency equation is satisfied:

$$\varepsilon = \sum_{n=1}^N \frac{\nu k_1^2 n^2}{\alpha_n}. \tag{30}$$

Then, once $\tilde{\Delta}_n(\mu)$ is known, one can determine $\tilde{\phi}_n(\mu)$ through equations (17) to (19).

In the following, we take the infinite N limit and consider the small viscosity regime in which developed turbulence is usually observed. To determine the energy spectrum, i.e., the values of α_n, we require that the energy flux ε does not depend on the viscosity in the limit $\nu \to 0$. In other words, the α_n must be such that the sum in equation (30) is independent of ν. Assuming that k_1 is small and that α_n depends smoothly on n, one can replace the above sum by an integral over the wavenumber $k = k_1 n$, introducing the energy density $E(k)$ at wavenumber k through $E(k_n) = 1/(k_1 \alpha_n)$:

$$\sum_{n=1}^\infty \frac{\nu k_1^2 n^2}{\alpha_n} = \sum_{n=1}^\infty \nu k_n^2 k_1 E(k_n) \approx \int_0^\infty \mathrm{d}k \, \nu k^2 E(k). \tag{31}$$

Making the assumption that the energy ρ_n has the dimension of a velocity square, it is well known from Kolmogorov's K41 theory [21] that a dimensionally consistent solution for $E(k)$ that yields a non-vanishing energy flux for small ν is given by

$$E(k) = \varepsilon^{\frac{2}{3}} k^{-\frac{5}{3}} g(\eta k), \qquad \eta \equiv \left(\frac{\nu^3}{\varepsilon} \right)^{\frac{1}{4}}, \tag{32}$$

where η is the dissipation scale, and $g(x)$ is a scaling function with a finite limit when $x \to 0$ and such that $\int_0^\infty \mathrm{d}x \, x^{1/3} g(x) = 1$. So our model allows one to construct a stochastic model for turbulence obeying the K41 phenomenology. Its main drawback is that due to the factorization condition, the energy spectrum must be somehow included by hand in the stochastic rules, whereas one may rather wish this spectrum to arise as a solution of the model, by including in the microscopic rules only some basic physical principles. In this respect, it may be of great interest to be able to go beyond the factorized steady-state distribution, and one might try to solve this model with a more general matrix product ansatz.

Conclusion

In the present note, we have introduced a class of exactly solvable stochastic models, which include both transport and dissipation phenomena. The factorization condition for the probability distribution, as well as an explicit solution for the latter, has been derived. Some specific models within the class may be used to illustrate the appearance of (generalized) Gumbel distributions in complex systems [20], or to model, in a schematic way, the cascade phenomenon in turbulent flows. More generally, one might hope that this class of models could help to understand some fundamental issues concerning the statistical behaviour of dissipative systems with a large number of degrees of freedom.

Acknowledgments

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